EUPAC Seminar Series

1-Form Symmetries and Confinement

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October 3, 2025

The University of Queensland

The Goal: A Symmetry for Confinement

- Following on from Jonathan's introduction to 0-form symmetries, we want to apply this language to a specific problem: confinement.
- The classic "area law" criterion for confinement, while useful, has limitations (non-local, string breaking).
- **Goal:** Can we generalise the notion of symmetry itself to find a more fundamental description of confinement?

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1. Generalise from 0-Form to p-Form Global Symmetries
2. Specialise to 1-Form Symmetries in Gauge Theory
3. Connect to Confinement
4. (If time permits) A Worked Example in $SU(2)$

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Generalising Symmetries

Recap and Generalisation

- As we saw in the previous talk, a **0-form symmetry** is characterised by:
  - Charged operators are points (0-dimensional).
  - The charge is defined on a codimension-1 surface.

- **Generalisation:** A p-**form symmetry** is characterised by:
  - Charged operators are p-dimensional surfaces.
  - p-form symmetries are codimension-$(p+1)$ invertible, topological operators.

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Today, we are interested in the next simplest case: 1-form symmetries.

1-Form Symmetries and Non-Locality

- For a **1-form symmetry**, the charged objects are lines (1-dimensional), like a Wilson line:

$$W_R(C) = \mathrm{Tr}_R P \exp(i \oint_C A_\mu dx^\mu)$$

This operator is non-local: its value depends on the gauge field A_μ at every point along the entire path C.

- The charge operator $U_g(\Sigma_{d-2})$ acts on a line operator $W(C)$ when they link. This topological condition ensures compatibility with Noether's theorem.

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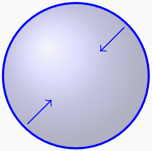
Topological Nature of Charge Operators

0-Form Analogy A charge operator $U_g(\Sigma_{d-1})$ on a sphere can be shrunk. If it encloses no charge, it shrinks to a point (gives Identity).



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1-Form Case A charge operator $U_g(\Sigma_{d-2})$ can be deformed. If it doesn't *link* with a charged line, it can be shrunk away (gives Identity).

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Linking is the key topological obstruction for 1-form symmetries.

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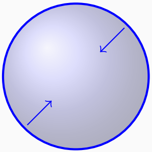

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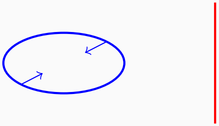

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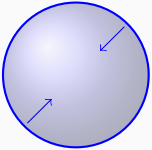

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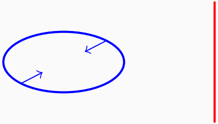

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Abelian Nature of Higher Symmetries

- A key feature of $p > 0$-form symmetries is that their symmetry group must be Abelian.
- This can be understood topologically: codimension-1 charge surfaces can always be ordered and slid past one another.

- Surfaces of codimension > 1 can be topologically linked, but the algebra of operators remains commutative.
- This means that even when the underlying gauge group G is non-Abelian (like $SU(N)$), the resulting 1-form symmetry group (its centre $Z(G)$) must be Abelian.

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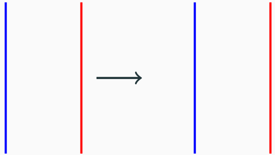

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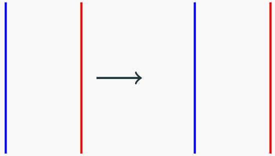

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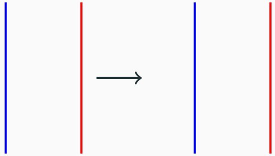

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1-Form Symmetries in Gauge Theory

Constructing the Charge Operator

How do we build U_g from a conserved current?

- Start with a conserved current. For a 1-form symmetry, this is a 2-form current J satisfying $d \star J = 0$.
- Define the conserved charge Q as the flux of this current through a codimension-2 surface Σ_{d-2}:

$$Q(\Sigma_{d-2}) = \int_{\Sigma_{d-2}} \star J$$

- The symmetry operator U_g is the exponential of this charge:

$$U_g(\Sigma_{d-2}) = \exp\left(i\alpha Q(\Sigma_{d-2})\right)$$

  (for some normalisation α).
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Goal: Apply this to Maxwell theory in d dimensions.

Setup: The field is the 2-form field strength $\boldsymbol{F} = d\boldsymbol{A}$. The equations of motion in a source-free region are:

$$d\boldsymbol{F} = 0 \quad \text{(Bianchi Identity)}$$
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We can now treat both F and $\star F$ as conserved currents to generate symmetries.

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The Magnetic 1-Form Symmetry

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- This is a mathematical identity (topological). It holds off-shell.
- We can treat $\boldsymbol{F}$ as a conserved 2-form current. The charged objects are magnetic lines.
- (More formally, this symmetry relates to topological classes of the gauge bundle, but $d\boldsymbol{F} = 0$ provides the direct physical picture.)

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Summary for General Gauge Theories

For a general gauge group G (like $SU(N)$):

- **Magnetic 1-Form Symmetry:** Typically $U(1)$. Charged objects are magnetic lines.
- **Electric 1-Form Symmetry:** This is the important one for confinement.
  - It corresponds to the centre of the gauge group, $Z(G)$. For $SU(N)$, this is $\mathbb{Z}_N$. This means the gauge group for a 1-form symmetry is coarse-grained, in the sense that the symmetry only sees charges up to N-ality.
  - The objects charged under this symmetry are the representations of Wilson lines with nontrivial N-ality.

The point being: We have a symmetry that acts directly on Wilson lines, which allows us to use symmetry principles to study confinement.

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Confinement

Confinement and the Electric 1-Form Symmetry

The expectation value $\langle W(C) \rangle$ tells us how the vacuum responds to the insertion of charged Wilson lines.

Case 1: Symmetry is preserved

- The vacuum is disordered.
- Wilson lines are suppressed.
- $\langle W(C) \rangle \sim \exp(-\text{Area}(C))$
- Confinement

Case 2: Symmetry is broken

- The vacuum is ordered.
- Wilson lines condense.
- $\langle W(C) \rangle \sim \exp(-\text{Perimeter}(C))$
- Deconfined / Higgs phase

Confinement is the phase where the electric 1-form symmetry is preserved.

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Physical Picture: Why Does This Work?

- What does a "disordered vacuum" (preserved symmetry) mean physically?
- It means the vacuum is a fluctuating condensate of objects charged under the magnetic 1-form symmetry.
- The vacuum is a sea of virtual magnetic loops (monopole world-surfaces).

When the Wilson loop is inserted, the fluctuating magnetic loops link with it, causing its phase to decohere. The bigger the area, the more decoherence $\implies$ Area Law.

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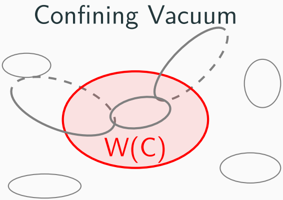

When the Wilson loop is inserted, the fluctuating magnetic loops link with it, causing its phase to decohere. The bigger the area, the more decoherence $\implies$ Area Law.

Physical Picture: Why Does This Work?

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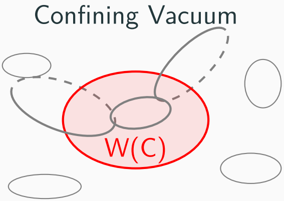
- The vacuum is a sea of virtual magnetic loops (monopole world-surfaces).



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A Worked Example

Operator Action via Deformation

- The action of the topological operator U_g on W_R can be seen by deforming/sliding the operator off the line.
- This is equivalent to cutting the loop, inserting a group element $g \in Z(G)$, and rejoining.

For an irrep R, this multiplies the operator by a phase:

$$W'_R(\gamma) = \frac{\mathrm{Tr}_R(g)}{\mathrm{Tr}_R(\mathrm{Id})} W_R(\gamma)$$

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Worked Example: $SU(2)$ Spin-s Representation

Problem: Take $G = SU(2)$ and let R_s be the spin-s representation. What is the phase obtained for the Wilson line $W(s)$ from the non-trivial centre element?

- The centre is $Z(SU(2)) = \{+I, -I\} \simeq \mathbb{Z}_2$. The non-trivial element is $g = -I$.
- The phase is given by the formula $\frac{\mathrm{Tr}_s(-I)}{\mathrm{Tr}_s(I)}$.
- The spin-s representation has dimension $2s + 1$. So, the identity matrix has trace $\mathrm{Tr}_s(\pm I) = \pm(2s + 1)$.
- The phase is given by $(-1)^{2s}$.
    - For **integer spins** ($s \in \{0, 1, \ldots\}$), $2s$ is even, phase is $+1$. (Uncharged)
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Result

Only Wilson lines in half-integer spin representations are charged under the $\mathbb{Z}_2$ 1-form symmetry.

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